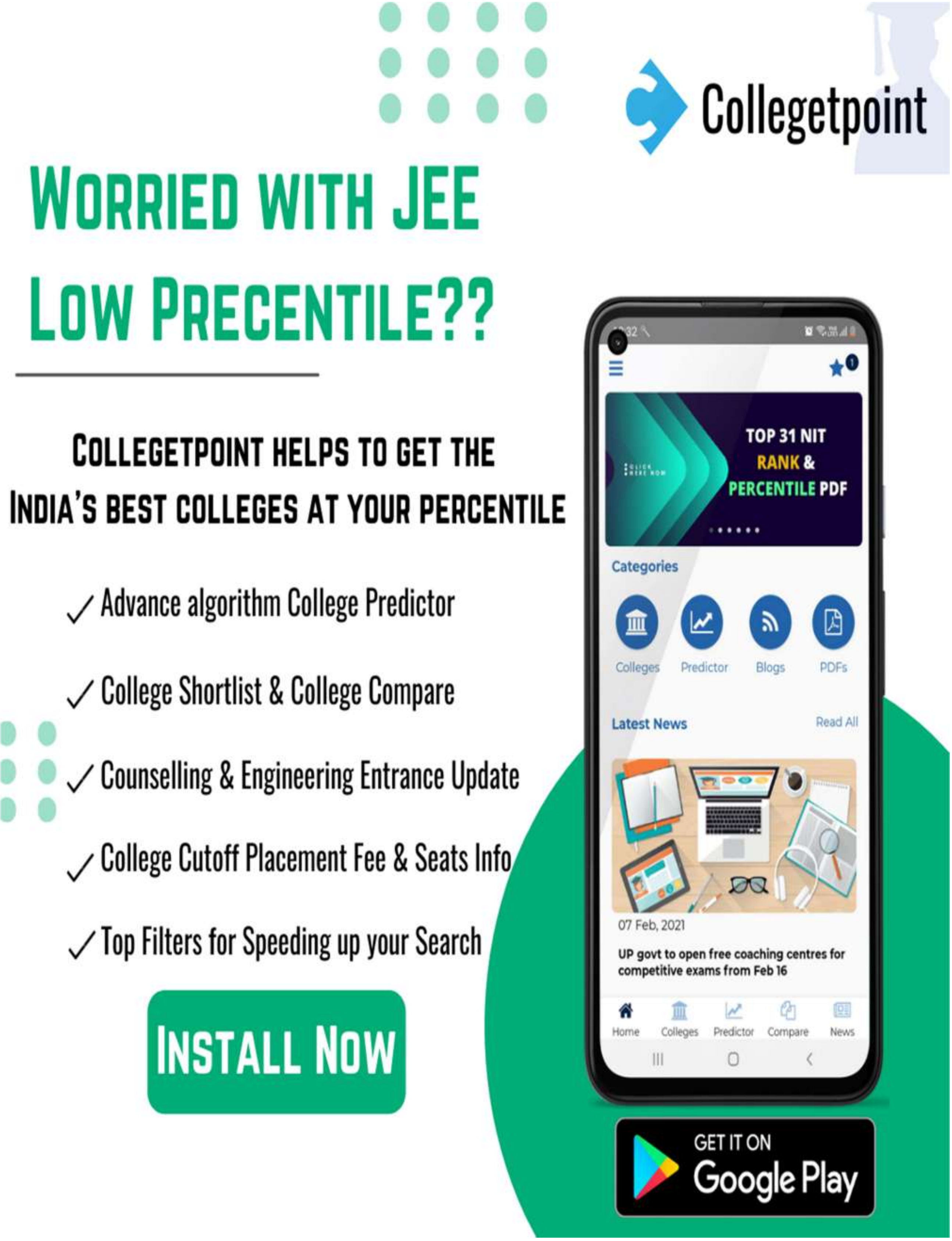
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Manipal Entrance Test (MET) 2021 Syllabus

MET 2021 Test Details

- Mode: Online Remote Proctored Test (RPT)
- Test Duration: **120** minutes
- Total Questions: **60**
    - **Multiple Choice Questions (MCQs):** Mathematics (15), Physics (10), Chemistry (10), and English (10)
    - **Numerical Answer Type (NAT):** Mathematics (5), Physics (5), and Chemistry (5)
- Max Marks: **240**
- Marking Scheme
    - **MCQ: +4** for every correct answer, **-1** for every wrong answer, **0** for every unanswered question
    - **NAT: +4** for every correct answer, **0** for every wrong, **0** for every unanswered question
- MET 2021 Schedule: June 11 – 14, 2021
    - There will be ONE attempt only

MATHEMATICS

Sets and Functions

Sets: Sets and their representations, The empty set, Finite and infinite sets, Equal sets, Subsets, Power set, Universal set, Venn Diagrams, Operation on Sets, Complement of a set, Union and intersection of two sets, **Relations and Functions:** Cartesian product of sets, relations and functions, **Trigonometric Functions:** Angles, Trigonometric functions, Trigonometric functions of sum and difference of two angles and trigonometric equations.

Algebra

Mathematical Induction: Motivation, Principle of mathematical induction, **Numbers and Quadratic Equations:** Complex numbers, Algebra of complex numbers, The modulus and the conjugate of a complex number, Argand plane and polar representation and Quadratic equations, **Linear Inequalities:** Inequalities, Algebraic solutions of linear inequalities, graphical solution and solution of system of linear inequalities in two variables, **Permutations and Combinations:** Permutations and Combinations, Derivation of formulae and their connections**, Binomial Theorem:** Binomial theorem for positive integral indices, General and middle terms, Applications, **Sequence and Series:** Sequences, Series, Arithmetic progression, Geometric progression, Relationship between AM and GM, sum to n terms of special series, **Matrices:** Matrix, types, operations, Transpose, Symmetric and skew symmetric matrices, elementary operation, Invertible matrices, **Determinants:** Properties, Area of a triangle, minors and cofactors, Adjoint and inverse of a matrix, applications

Coordinate Geometry

Straight Lines: Slope of a line, Various forms of the equation of a line, General equation of a line, Distance of a point from a line, **Conic Sections:** Section of a cone, Circle, Parabola, Ellipse, Hyperbola, **Introduction to Three-dimensional Geometry:** Coordinate axes and coordinate planes in three-dimensional space, Coordinates of a point in space, Distance between two points, Section formula

Calculus

Limits and Derivatives: Intuitive idea of derivatives, Limits of trigonometric functions, Derivatives**, Continuity and Differentiability:** Exponential and logarithmic functions, logarithmic differentiation, Derivatives of functions in parametric forms, Second order derivative, Mean value theorem, **Applications of Derivatives:** Rate of change of quantities, Increasing and decreasing functions, Tangents and normals, Approximations, Maxima and Minima, **Integrals and their Application:** Integration, methods of integration, Integrals of some particular functions, Integration by partial fractions, Integration by parts, Definite Integral, Fundamental theorem of calculus, Evaluation of definite integrals by substitution, Properties of definite integrals, Area under simple curves and two curves, **Differential equations:** Basic concepts, General and particular solutions of a differential equation, formation of a differential equation, methods of solving first order, first degree differential equations

Mathematical Reasoning: Statements, New statements from old, Special words/phrases, Implications, Validating statements, **Statistics and Probability:** Measure of dispersion, Range, Mean deviation, Variance and standard deviation, Analysis of frequency distributions, Random experiments, Event, Axiomatic approach to probability, Conditional probability, multiplication theorem on probability, Independent events, Bayes' theorem, Random variables and its probability distributions, Bernoulli trials and binomial distribution, **Relations and Functions:** Types of relations and functions, Composition of functions and invertible function, Binary operations, **Inverse Trigonometric Functions:** Basic concepts, Properties of inverse trigonometric functions, **Vector algebra:** Basic concepts, types of vectors, Addition of vectors, Multiplication of a vector by a scalar, Product of two vectors, **Three-Dimensional Geometry:** Direction cosines and direction ratios of a line, Equation of a line in space, Angle between two lines, Shortest distance between two lines, Plane, Coplanarity of two lines, Angle between two planes, Distance of a point from a plane, Angle between a line and a plane, **Linear programming:** Problem and its mathematical formulation, different types of linear programming problems

Mathematics

66. The solution of the equation. $\log\left(\log_5\left(\sqrt{x+5}+\sqrt{x}\right)\right)=0$ is

(a) 2 (b) 4 (c) 3 (d) 8

67. Let $\frac{1}{q+r}$, $\frac{1}{r+p}$ and $\frac{1}{p+q}$ are in A.P. where $p, q, r, \neq 0$, then

(a) p, q, r are in A.P.

(b) p^2, q^2, r^2 are in A.P.

(c) $\frac{1}{p}\cdot\frac{1}{q}\cdot\frac{1}{r}$ in A.P.

(d) none of these

68. If $b \in R^+$ then the roots of the equation $(2+b)x^2+(3+b)x+(4+b)=0$ is

(a) real and distinct (b) real and equal (c) imaginar (d) cannot predicted

69. Solve for integral solutions $x_1 + x_2 + x_3 + \ldots + x_6 \leq 17$, where $1 \leq x_i \leq 6$, $i = 1, 2, \ldots 6$.
Number of solutions will be

(a) ${}^{17}C_6 - 6\,{}^{11}C_5$ (b) ${}^{17}C_{11} - 6\,{}^{11}C_5$ (c) ${}^{17}C_5 - 6\,{}^{11}C_5$ (d) ${}^{17}C_{11} - 5\,{}^{11}C_6$

70. The probability that a certain beginner at golf gets a good shot if he uses the correct club is $\frac{1}{3}$, and the probability of a good shot with an incorrect club is $\frac{1}{4}$. In his bag there are 5 different clubs, only one of which is correct for the shot in question. If he chooses a club at random and take a stroke, the probability that he gets a good shot is

(a) $\frac{1}{3}$ — — —

(b) 12

(c) 15

(d)

$\frac{1}{2}$

71. OPQR is a square and M, N are the middle points of the side PQ and QR respectively. Then the ratio of the area of the square and the triangle OMN is

(a) 4 : 1 (b) 2 : 1 (c) 4 : 3 (d) 8 : 3

72. Two vertices of an equilateral triangle are (– 1, 0) and (1, 0) and its circumcircle is

(a) $x^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}$ (b) $x^2 - \left(y + \frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3}$

(c) $x^2 + \left(y - \frac{1}{\sqrt{3}}\right)^2 = -\frac{4}{3}$ (d) none of these

73. If in a ΔABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is always

(a) isosceles triangle (b) right angled (c) acute angled (d) obtuse angled

74. If the vertex and the focus of a parabola are (–1, 1) and (2, 3) respectively, then the equation of the directrix is

(a) $3x + 2y - 25 = 0$ (b) $x + 2y + 7 = 0$ (c) $2x - 3y + 10 = 0$ (d) $3x + 2y + 14 = 0$.

75. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having its centre at (0, 3) is

(a) 4 (b) 3 (c) $\sqrt{12}$ (d) 7/2

76. If $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$ are four concyclic points on the rectangular hyperbola $xy = c^2$, then the co-ordinates of the orthocentre of ΔPQR are

(a) $(x_4, -y_4)$ (b) (x_4, y_4) (c) $(-x_4, -y_4)$ (d) $(-x_4, y_4)$

77. The coefficient of $x^n y^n$ in the expansion of $[(1 + x)(1 + y)(x + y)]^n$ is

(a) $\sum_{r=0}^{n} C_r$ (b) $\sum_{r=0}^{n} C_r^{\,2}$ (c) $\sum_{r=0}^{n} C_r^{\,3}$ (d) none of these

78. z_0 is one of the roots of the equation $z^n \cos\theta_0 + z^{n-1}\cos\theta_1 + \ldots + \cos\theta_n = 2$, where $\theta_i \in R$, then

(a) $|z_0| < \frac{1}{2}$ (b) $|z_0| > \frac{1}{2}$ (c) $|z_0| = \frac{1}{2}$ (d) none of these

79. The second order differential equation is

(a) $y'^2 + x + y^2$ (b) $y'y'' + y = \sin x$ (c) $y''' + y'' + y = 0$ (d) $y' = 0$

80. $\int e^{3x}\left(\frac{1+3\sin x}{1+}\right)dx$ is equal to

(a) $e^{3x}\cot x + c$ (b) $e^{3x}\tan\frac{x}{2} + c$ (c) $e^{3x}\sin x + c$ (d) $e^{3x}\cos x + c$

81. If m and n are positive integers and $f(x) = \int_1^x (t-a)^{2n}(t-b)^{2m+1}\,dt$, $a \neq b$, then

(a) $x = b$ is a point of local minimum (b) $x = b$ is a point of local maximum

(c) $x = a$ is a point of local minimum (d) $x = a$ is a point of local maximum

82. If in a triangle ABC $\frac{2\cos A}{a} + \frac{\cos B}{b} + \frac{2\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then the value of the angle A is

(a) 45° (b) 90° (c) 135° (d) 60°

83. The general solution of the equation $2^{\cos 2x} + 1 = 3.2^{-\sin^2 x}$ is

(a) $n\pi$ (b) $\left(n + \frac{1}{2}\right)\pi$ (c) $\left(n - \frac{1}{2}\right)\pi$ (d) all of the above.

84. Total number of positive real values of x satisfying $2[x] = x + \{x\}$, where [.] and {.} denote the greatest integer function and fractional part respectively is equal to

(a) 2 (b) 1 (c) 0 (d) 3

85. If $\lim_{x\to 0}\frac{((a-n)nx - \tan x)\sin nx}{x^2} = 0$, where n is nonzero real number, then a is equal to

(a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n + \frac{1}{n}$

86. $f(x) = \begin{cases} 4x - x^3 + \ln\left(a^2 - 3a + 3\right), & 0 \le x < 3 \\ x - 18, & x \ge 3 \end{cases}$. Find the complete set of values of a such that $f(x)$ has a local minima at $x = 3$ is

(a) $[-1, 2]$ (b) $(-\infty, 1) \cup (2, \infty)$ (c) $[1, 2]$ (d) $(-\infty, -1) \cup (2, \infty)$

87. The number of values of k for the system of equations $(k+1)x + 8y = 4k$ and $kx + (k+3)y = 3k - 1$ has infinitely many solutions

(a) 0 (b) 1 (c) 2 (d) infinite

88. The matrix $\begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}$ is

(a) unitary (b) null matrix (c) symmetric (d) none of these

89. The area between the curves $y = xe^{x}$ and $y = xe^{-x}$ and the line $x = 1$ is

(a) $2e$ (b) e (c) $2/e$ (d) $1/e$

90. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ and $|\vec{a} - \vec{b}| < 1$ then θ lies in the interval

(a) $\left[0, \frac{\pi}{6}\right)$ (b) $\left(\frac{5\pi}{6}, 2\pi\right]$ (c) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$ (d) $\left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

Chap-1 - Sets, Relations and Functions

Formulae Chart

- If A is a set with $n(A) = m$ then,
  a. Numbe of subsets of $A = 2^m$
  b. Number of proper subsets $= 2^m - 1$
- If A and B are finite sets then
  $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- If A, B, C are finite sets, then
  $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.
- If $n(A) = n$
  a. Number of identity relation in set $A = 1$.
  b. Number of reflexive relation in set $A = 2^{n^2-n}$.
- $n(A) = m$, $n(B) = n$
  a. Number of relations from A to $B = 2^{mn}$
  b. Number of functions from A to $B = n^m$
  c. Number of one-one functions
     (i) If $m > n$, then number of one-one function = 0
     (ii) If $n \ge m$, then number of one-one function $= {}^nP_m$

Chap-02 - Trigonometric Functions

Formulae Chart

Summation/Continued Product

$$\sin\theta + \sin(\theta+\phi) + \sin(\theta+2\phi) + \ldots\ \text{to } n \text{ terms} = \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\sin\left(\theta + \frac{n-1}{2}\phi\right) \quad (\phi \ne 2m\pi, n \in Z)$$

$$= \frac{\sin n\left(\frac{\text{difference}}{2}\right)}{\sin\frac{\text{difference}}{2}}\sin\left\{\frac{\text{first angle} + \text{last angle}}{2}\right\}$$

$$\cos\theta + \cos(\theta+\phi) + \cos(\theta+2\phi) + \ldots\ \text{to } n \text{ terms} = \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\cos\left(\theta + \frac{n-1}{2}\phi\right) \quad (\phi \ne 2m\pi, n \in Z)$$

$$= \frac{\sin n\left(\frac{\text{difference}}{2}\right)}{\sin\frac{\text{difference}}{2}}\cos\left\{\frac{\text{first angle} + \text{last angle}}{2}\right\}$$

$$\cos A\cos 2A\cos 2^2A\cos 2^3A \ldots \cos 2^{n-1}A = \frac{1}{2^n\sin A}(\sin 2^nA) \quad (A \ne n\pi, n \in Z)$$

$\sqrt{2+\sqrt{2+\sqrt{2+\ldots+\sqrt{2+2\cos 2^nA}}}} = 2\cos A, \forall n \in N$, where there are n square root sign on L.H.S.

$\tan A + 2\tan 2A + 2^2\tan 2^2A + \ldots + 2^n\tan 2^nA + 2^{n+1}\cot 2^{n+1}A = \cot A, \forall n \in N.$

- The solution of trigonometric equations of the form $a\sin\theta \pm b\cos\theta = c$ exists iff $\left|\frac{c}{\sqrt{a^2+b^2}}\right| \le 1$.

Napier's Analogy or Tangent Rule

In any ΔABC,

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$

$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$

Half-angle formulae

In any ΔABC,

$\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$

$\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$

$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{(s-b)(s-c)}{\Delta}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}} = \frac{(s-c)(s-a)}{\Delta}$

$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \frac{(s-a)(s-b)}{\Delta}$

$\cot\frac{A}{2} = \sqrt{\frac{s(s-a)}{(s-b)(s-c)}} = \frac{s(s-a)}{\Delta}$, $\cot\frac{B}{2} = \sqrt{\frac{s(s-b)}{(s-c)(s-a)}}$

$= \frac{s(s-b)}{\Delta}$, $\cot\frac{C}{2} = \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{s(s-c)}{\Delta}$

Chap-03 - Inverse Trigonometric Functions

Formulae Chart

Inverse Trigonometric Function

1.

A	B
(i) $\sin^{-1}\sin\theta = \theta$, for all $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	(i) $\sin(\sin^{-1}x) = x$, $x \in [-1, 1]$
(ii) $\cos^{-1}\cos\theta = \theta$, for all $\theta \in [0, \pi]$	(ii) $\cos(\cos^{-1}x) = x$, $x \in [-1, 1]$
(iii) $\tan^{-1}\tan\theta = \theta$, for all $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	(iii) $\tan(\tan^{-1}x) = x$, $x \in R$

2. (a) (i) $\sin^{-1}(-x) = -\sin^{-1}(x)$, $x \in [-1, 1]$
   (ii) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$, $x \in [-1, 1]$
   (iii) $\tan^{-1}(-x) = -\tan^{-1}x$, $x \in R$
3. (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy < 1$
   $= \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, if $xy > 1$, $x > 0$, $y > 0$
   $= \tan^{-1}\left(\frac{x+y}{1-xy}\right) - \pi$, if $xy > 1$, $x < 0$, $y < 0$
   (ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $xy > -1$
   $= \tan^{-1}\left(\frac{x-y}{1+xy}\right) - \pi$, if $x < 0$, $y > 0$ & $xy < -1$
   $= \pi + \tan^{-1}\left(\frac{x-y}{1+xy}\right)$, if $x > 0$, $y < 0$, $xy < -1$

Note : If $x_1, x_2, x_3, \ldots, x_n \in R$, then

$$\tan^{-1}x_1 + \tan^{-1}x_2 + \ldots + \tan^{-1}x_n = \tan^{-1}\left(\frac{S_1 - S_3 + S_5 - S_7 + \ldots}{1 - S_2 + S_4 - S_6 + \ldots}\right)$$

where S_k denotes the sum of products of $x_1, x_2, \ldots, x_n$ taken k at a time.

Chap-04 - Complex Numbers & Quadratic Equations

Formulae Chart

1. Conjugate of a complex number $z = a + ib$ is denoted by $\bar{z}$ and is equal to $a - ib$.
   For any three complex numbers z, z_1 and z_2,
   (i) $z\bar{z} = (\text{Re } z)^2 + (\text{Im } z)^2$
   (ii) If z is purely real then $z = \bar{z}$
   (iii) If z is purely imaginary then $z = -\bar{z}$
   (iv) $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
   (v) $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$
   (vi) $\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$
   (vii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$, $z_2 \ne 0$
2. The modulus of a complex number $z = a + ib$ is denoted by $|z|$ and is equal to $\sqrt{a^2+b^2}$
   For any three complex numbers z, z_1 and z_2,
   (i) $|z| = 0$ iff $z = 0$
   (ii) $z\bar{z} = |z|^2$ or $z^{-1} = \frac{\bar{z}}{|z|^2}$, $z \ne 0$
   (iii) $|z| = |\bar{z}| = |-z|$
   (iv) $-|z| \le \text{Re } z \le |z|$ and $-|z| \le \text{Im } z \le |z|$
   (v) $|z_1 z_2| = |z_1|\,|z_2|$
   (vi) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \ne 0$
   (vii) $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$
   (viii) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\text{Re}(z_1\bar{z}_2)$
   (ix) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\text{Re}(z_1\bar{z}_2)$

3. If $z = a + ib$ is a complex number, then

$$\sqrt{a+ib} = \begin{cases} \pm\left[\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)}\right) + i\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)}\right)\right], & \text{if } b \text{ is positive} \\ \pm\left[\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}+a\right)}\right) - i\left(\sqrt{\frac{1}{2}\left(\sqrt{a^2+b^2}-a\right)}\right)\right], & \text{if } b \text{ is negative} \end{cases}$$

4. a. If $ax^2 + bx + c = 0$, $a \ne 0$ is a quadratic equation with real coefficients, then its roots are given by
   $\alpha, \beta = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ or $\alpha, \beta = \frac{-b \pm \sqrt{D}}{2a}$
   where $D = b^2 - 4ac$ is the discriminant of the equation. If $D < 0$, then the roots are imaginary and are given by $\alpha = \frac{-b \pm i\sqrt{4ac-b^2}}{2a}$ (i.e., imaginary roots occur conjugate pairs)
   b. D = a perfect square, then roots of $ax^2 + bx + c = 0$ are rational only when the coefficients a, b, c are rationals otherwise roots are irrational.
   Irrational roots of an equation always occur in conjugate pairs, i.e., if $p + \sqrt{q}$ is one root of $ax^2 + bx + c = 0$, then other root must be $p - \sqrt{q}$.
   c. If roots of $ax^2 + bx + c = 0$ are equal in magnitude and opposite in signs, then sum of roots $= 0 = -\frac{b}{a}$
   $\Rightarrow$ b = coefficient of $x = 0$.
   d. To have both roots positive of $ax^2 + bx + c = 0$, $D > 0$, sum of roots > 0, product of roots > 0.
   e. To have both roots negative of $ax^2 + bx + c = 0$, $D > 0$, sum of roots < 0, product of roots > 0.
   f. $ax^2 + bx + c > 0$ for all $x \in R$, $a > 0$, $D < 0$.
   g. $ax^2 + bx + c < 0$ for all $x \in R$, $a < 0$, $D < 0$.

Chap-05 - Sequences and Series

Formulae Chart

1. For Arithmetic progressions :
   (i) $a_n = a + (n-1)d$
   (ii) $d = \frac{b-a}{n-1}$, if b is the n^{th} term
   (iii) $S_n = \frac{n}{2}[2a + (n-1)d]$
   $= \frac{n}{2}[a + l]$
   Where a = first term
   d = common difference
   n = number of terms
   l = last term
2. For Geometric progressions :
   (i) $a_n = ar^{n-1}$
   (ii) $r = \left(\frac{b}{a}\right)^{\frac{1}{n-1}}$, if b is the n^{th} term
   (iii) $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$, $r \ne 1$
   (iv) $S_\infty = \frac{a}{1-r}$ provided $|r| < 1$ i.e. $-1 < r < 1$
   Where a = first term
   r = common ratio
   b = n^{th} term
   n = number of terms
3. For two positive numbers a and b
   A.M. $\ge$ G.M.
   Sum to n terms of special series :
   (i) Sum of first n natural numbers $= \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
   (ii) Sum of squares of the first n natural numbers $= \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
   (iii) Sum of cubes of the first n natural numbers $= \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$

Chap-06 - Permutation and Combination

Formulae Chart

1. ${}^nP_r = \frac{n!}{(n-r)!}$, where $0 \le r \le n$
2. The number of permutation of n different things taken r at a time, where repetition is allowed is n^r.
3. The number of permutations of n distinct things, taken all at a time is $n!$.
4. The number of permutations of n objects taken all at a time, where k_1 are alike of one kind, k_2 are alike of second kind, k_3 are alike of third kind, ………, k_r are alike of r^{th} kind such that $k_1 + k_2 + k_3 + \ldots + k_r = n$ is $\frac{n!}{k_1!\,k_2!\,k_3!\ldots k_r!}$
5. ${}^nC_r = \frac{n!}{r!(n-r)!}$, where $0 \le r \le n$
6. ${}^nP_r = {}^nC_r\, r!$
7. ${}^nC_r = {}^nC_{n-r}$, where $0 \le r \le n$
8. If ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$
9. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, where $1 \le r \le n$
10. $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$, $r \ge 1$

Chap-07 - Binomial Theorem

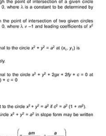
Formulae Chart

1. The general term (T_{r+1}) in the expansion of $(x+a)^n$ is given by $T_{r+1} = {}^nC_r\, x^{n-r}a^r$.
2. The middle term(s) in the expansion of $(x+a)^n$ is/are
   (i) $T_{\frac{n}{2}+1}$ if n is even
   (ii) $T_{\frac{n+1}{2}}$, $T_{\frac{n+3}{2}}$ if n is odd.
3. Among the binomial coefficients, the greatest binomial coefficients is
   (i) ${}^nC_{\frac{n}{2}}$ if n is even (ii) ${}^nC_{\frac{n+1}{2}} = {}^nC_{\frac{n-1}{2}}$ if n is odd.
4. ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$.
5. (a) ${}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 + \ldots + {}^nC_n = 2^{n-1} = {}^nC_1 + {}^nC_3 + \ldots + {}^nC_{n-1}$, if n is even.
   (b) ${}^nC_0 + {}^nC_2 + {}^nC_4 + \ldots + {}^nC_{n-1} = 2^{n-1} = {}^nC_1 + {}^nC_3 + \ldots + {}^nC_n$, if n is odd.

Chap-08 - Straight Lines

Formulae Chart

1. **Straight line**
   (i) Every equation of first degree in x and y always represents a straight line.
   (ii) The equation of straight line passing through (x_1, y_1) and making an angle θ with positive direction of x-axis can be written parametrically as
   $\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$
   Any point on this distant r from (x_1, y_1) is
   $(x_1 \pm r\cos\theta, y_1 \pm r\sin\theta)$



   (iii) The equation of straight line passing through two given points (x_1, y_1) and (x_2, y_2) is
   $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
   i.e. $\frac{x - x_1}{x_1 - x_2} = \frac{y - y_1}{y_1 - y_2}$
   (iv) The length of perpendicular drawn from (x_1, y_1) to the straight line $ax + by + c = 0$ is
   $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}}\right|$
   Distance between two parallel lines $ax + by + c = 0$ and $ax + by + c_1 = 0$ is
   $\left|\frac{c - c_1}{\sqrt{a^2+b^2}}\right|$
   (v) The equation of straight line passing through the point of intersection of two given lines $L_1 = 0$ and $L_2 = 0$ may be written as
   $L_1 + KL_2 = 0$
   (vi) Relative position of the points (x_1, y_1) and (x_2, y_2) with respect to the line $ax + by + c = 0$, the points (x_1, y_1) and (x_2, y_2) lie on the same side or opposite side of the line $ax + by + c = 0$ according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same sign or opposite sign.
   (vii) The equations of the bisector of the angles between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ $(a_1b_2 \ne a_2b_1)$ are given by
   $\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$

3. (i) The foot of the perpendicular (α, β) from (x_1, y_1) to the straight line $ax + by + c = 0$ is given by
   $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$
   (ii) The image or reflection of the point (x_1, y_1) in the line $ax + by + c = 0$ is (α, β) where
   $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$

Chap-09 - Circle

Formulae Chart

1. **Equation of Circle**
   (i) The equation of the circle on the line segment joining (x_1, y_1) and (x_2, y_2) as diameter is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
   (ii) The equations of the circle with centre at (h, k) and touching x-axis and y-axis are
   $x^2 + y^2 - 2hx - 2ky + h^2 = 0$ and $x^2 + y^2 - 2hx - 2ky + k^2 = 0$ respectively.
   (iii) The equation of the circle touching both axes and radius 'a' is
   $x^2 + y^2 \pm 2ax \pm 2ay + a^2 = 0$
   (iv) The equation of a circle passing through the point of intersection of a given circle $S = 0$ and a given line $L = 0$ is $S + \lambda L = 0$, where λ is a constant to be determined by the given condition.
   (v) The equation of a circle passing through the point of intersection of two given circles $S_1 = 0$ and $S_2 = 0$ is given by $S_1 + \lambda S_2 = 0$, where $\lambda \ne -1$ and leading coefficients of x^2 and y^2 in S_1 and S_2 are unity.
2. **Equations of Tangent and Normal**
   (i) The equation of the tangent and the normal to the circle $x^2 + y^2 = a^2$ at (x_1, y_1) is
   $xx_1 + yy_1 = a^2$ and $\frac{x}{x_1} - \frac{y}{y_1} = 0$ respectively.
   (ii) The equation of the tangent and the normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$
   and $\frac{x - x_1}{x_1 + g} = \frac{y - y_1}{y_1 + f}$ respectively.
   (iii) The straight line $y = mx + c$ will be tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$.
   Then the equation of the tangent to the circle $x^2 + y^2 = a^2$ in slope form may be written as
   $y = mx \pm a\sqrt{1 + m^2}$ and the points of contact are $\left(\mp\frac{am}{\sqrt{1+m^2}}, \pm\frac{a}{\sqrt{1+m^2}}\right)$
3. **Parametric Equation**
   (i) Parametric equation of circle $x^2 + y^2 = a^2$ is $x = a\cos\theta$, $y = a\sin\theta$.
   (ii) Parametric equation of circle $(x - x_0)^2 + (y - y_0)^2 = a^2$ is $x = x_0 + a\cos\theta$, $y = y_0 + a\sin\theta$.

Chap-10 - Conic Section

Formulae Chart

1. **Parabola**
   (i) Parametric equation of $y^2 = 4ax$ is $x = at^2$, $y = 2at$.
   (ii) Parametric equation of $x^2 = 4ay$ is $x = 2at$, $y = at^2$.
   (iii) Equation of tangent having slope m to $y^2 = 4ax$ is $y = mx + \frac{a}{m}$.
   (iv) Equation of tangent having slope m to $x^2 = 4ay$ is $y = mx - am^2$.
   (v) Equation of tangent at $(at^2, 2at)$ is $y = \frac{x}{t} + at$.
   (vi) Equation of normal having slope m is $y = mx - 2am - am^3$.
   (vii) Equation of normal at $(at^2, 2at)$ is $y = -tx + 2at + at^3$.
   (viii) $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are end points of focal chord if $t_1t_2 = -1$.
2. **Ellipse**
   (i) Parametric equation of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $x = a\cos\theta$, $y = b\sin\theta$.
   (ii) Equation of tangent having slope m to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{a^2m^2 + b^2}$.
   (iii) Equation of tangent at (x_1, y_1) is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$.
   (iv) Equation of tangent at $(a\cos\theta, b\sin\theta)$ is $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$.
   (v) Equation of normal at (x_1, y_1) is $\frac{x - x_1}{\frac{x_1}{a^2}} = \frac{y - y_1}{\frac{y_1}{b^2}}$.
   (vi) Equation of normal at $(a\cos\theta, b\sin\theta)$ is $ax\sec\theta - by\operatorname{cosec}\theta = a^2 - b^2$.
3. **Hyperbola**
   (i) Parametric equation of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x = a\sec\theta$, $y = b\tan\theta$.
   (ii) Equation of tangent having slope m to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $y = mx \pm \sqrt{a^2m^2 - b^2}$.
   (iii) Equation of tangent at (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

   (iv) Equation of tangent at $(a\sec\theta, b\tan\theta)$ is $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.
   (v) Equation of normal at (x_1, y_1) is $\frac{x - x_1}{\frac{x_1}{a^2}} = \frac{y - y_1}{-\frac{y_1}{b^2}}$.
   (vi) Equation of normal at $(a\sec\theta, b\tan\theta)$ is $ax\cos\theta + by\cot\theta = a^2 + b^2$.

Chap-11 Statistics

Formulae Chart

- Mean deviation for ungrouped data
  (a) about mean
  M.D. $(\bar{x}) = \frac{\sum|x_i - \bar{x}|}{n}$
  (b) about median
  M.D. $(M) = \frac{\sum|x_i - M|}{n}$
- Mean deviation for grouped data
  (a) about mean
  M.D. $(\bar{x}) = \frac{\sum f_i|x_i - \bar{x}|}{N}$
  (b) about median
  M.D. $(M) = \frac{\sum f_i|x_i - M|}{N}$, where $N = \sum f_i$
- Variance and standard deviation for ungrouped data
  $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$
  $\sigma = \sqrt{\frac{1}{n}\sum(x_i - \bar{x})^2}$
- Variance and standard deviation of a discrete frequency distribution.
  $\sigma^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2$, $\sigma = \sqrt{\frac{1}{N}\sum f_i(x_i - \bar{x})^2}$
- Variance and standard deviation of a continuous frequency distribution:
  $\sigma^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2$, $\sigma = \frac{1}{N}\sqrt{N\sum f_ix_i^2 - \left(\sum f_ix_i\right)^2}$
- Shortcut to find variance and standard deviation.
  $\sigma^2 = \frac{h^2}{N^2}\left[N\sum f_iy_i^2 - \left(\sum f_iy_i\right)^2\right]$
  $\sigma = \frac{h}{N}\sqrt{N\sum f_iy_i^2 - \left(\sum f_iy_i\right)^2}$, where $y_i = \frac{x_i - A}{h}$
- Co-efficient of variation (C.V.) $= \frac{\sigma}{\bar{x}} \times 100$, $\bar{x} \ne 0$.

Chap-12 - Matrices and Determinants

Formulae Chart

1. (i) **Trace of a square matrix**
   $\text{tr}A = \sum_{i=1}^{n} a_{ii} = a_{11} + a_{22} + \ldots + a_{nn}$
   where $= A = [a_{ij}]_{n \times n}$
   (ii) $\text{tr}(\lambda A) = \lambda\,\text{tr}A$
   (iii) $\text{tr}(A + B) = \text{tr}A + \text{tr}B$
   (iv) $\text{tr}(AB) = \text{tr}(BA)$
2. **Transpose of matrix (A)**
   (i) $(A^T)^T = A$
   (ii) $(A + B)^T = A^T + B^T$, A and B being of same size
   (iii) $(KA)^T = KA^T$, K is any complex number
   (iv) $(AB)^T = B^TA^T$, A and B being conformable for multiplication
3. **Conjugate of a matrix**
   (i) If $A = [a_{ij}]_{m \times n}$ then $\bar{A} = [\bar{a}_{ij}]_{m \times n}$
   (ii) $\overline{(\bar{A})} = A$
   (iii) $\overline{(A + B)} = \bar{A} + \bar{B}$
   (iv) $\overline{(KA)} = \bar{K}\bar{A}$, K being any complex number
   (v) $\overline{AB} = \bar{A}\,\bar{B}$, A and B being conformable for multiplication.
4. (i) If $A^T = A$, then A is symmetric matrix $[a_{ij} = a_{ji}]$
   (ii) If $A^T = -A$, then A is skew-symmetric matrix $[a_{ij} = -a_{ji}]$
   (iii) Every square matrix can be uniquely expressed as the sum of symmetric and skew-symmetric matrix.
   i.e. $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$
   $\frac{1}{2}(A + A^T) \to$ Symmetric matrix
   $\frac{1}{2}(A - A^T) \to$ Skew symmetric matrix
5. **Orthogonal matrix**
   If $AA^T = I$ where I is identity matrix where $A \to$ orthogonal matrix

6. **Adjoint of a matrix**
   $A = [a_{ij}]_{n \times n}$, a square matrix
   then $B = [A_{ij}]_{n \times n}$ where A_{ij} = cofactor of element a_{ij} in the determinant $|A|$
   then $\text{Adj}A = B^T$
   (i) $A(\text{adj}A) = (\text{adj}A) \cdot A = |A|\,I_n$, $I_n \to n$ rowed unit matrix
   (ii) $\text{adj}O = O$ and $\text{adj}\,I_n = I_n$, O being a zero matrix of order n
   (iii) $(\text{adj}A)^T = \text{adj}(A^T)$, $(\text{adj}A)^{-1} = \text{Adj}(A^{-1})$
   (iv) $\text{adj}(AB) = (\text{adj}B) \cdot (\text{adj}A)$
   (v) $\det(\text{adj}A) = (\det A)^{n-1}$, n being the order of matrix A
7. **Inverse of a matrix**
   If A be n row square matrix, such that $AB = BA = I_n$ then B is called the inverse of A, and A is said to be invertible.
   (i) $A^{-1} = \frac{1}{\det A}\text{adj}A = \frac{1}{|A|}\text{adj}A$, $|A| \ne 0$
   If $|A| = 0$, matrix is called singular
   If $|A| \ne 0$, matrix is called non-singular
   (ii) $(AB)^{-1} = B^{-1}A^{-1}$
   (iii) $(A^{-1})^T = (A^T)^{-1}$
   (iv) $(A^{-1})^n = (A^n)^{-1}$
   (v) $\det(A^{-1}) = (\det A)^{-1}$

DETERMINANTS

1. If a determinant is of order n then it contains n rows and n columns.
   Number of constituents in determinant $= n^2$
2. **Cramer's Rule :**
   (a) For Non-Homogeneous system of equation
   Let us consider a system of equations
   $a_1x + b_1y + c_1z = d_1$
   $a_2x + b_2y + c_2z = d_2$
   $a_3x + b_3y + c_3z = d_3$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

   $x = \frac{\Delta_1}{\Delta}$, $y = \frac{\Delta_2}{\Delta}$, $z = \frac{\Delta_3}{\Delta}$
   (i) If $\Delta \ne 0$, the system will have unique finite solution, so equation is consistent
   (ii) If $\Delta = 0$, at least any of $\Delta_1, \Delta_2, \Delta_3$ be non zero. The system has no solution i.e., equations are inconsistent.
   (iii) If $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$, the system will have infinite solution and at least one co-factor of Δ is non-zero. i.e., equations are consistent.
   (b) For Homogeneous system of equation
   $a_1x + b_1y + c_1z = 0$
   $a_2x + b_2y + c_2z = 0$
   $a_3x + b_3y + c_3z = 0$
   then $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, $\Delta_1 = \Delta_2 = \Delta_3 = 0$
   If $\Delta \ne 0$, system will have unique solution called trivial solution.
   If $\Delta = 0$, system will have infinite solution called non-trivial solution.

Chap-13- Limit, Continuity and Derivatives

Formulae Chart

1. **Properties of limits (these properties are used when either both limits are finite or both are indeterminate)**
   (i) $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
   (ii) $\lim_{x \to a} f(x) \cdot g(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
   (iii) $\lim_{x \to a}\frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \ne 0$
   (iv) $\lim_{x \to a}(kf(x)) = k\lim_{x \to a} f(x)$
   (v) $\lim_{x \to a}|f(x)| = \left|\lim_{x \to a} f(x)\right|$
   (vi) If $f(x) \le g(x)$, $\forall x$
   $\Rightarrow \lim_{x \to a} f(x) \le \lim_{x \to a} g(x)$
2. **Some standard results :**
   (i) $\lim_{x \to 0}\frac{\sin x}{x} = 1$, ($x$ in radians)
   (ii) $\lim_{x \to a}\frac{x^n - a^n}{x - a} = na^{n-1}$, $n \in Q$
   (iii) $\lim_{x \to 0}\frac{\tan x}{x} = 1$
   (iv) $\lim_{x \to 0}\frac{e^x - 1}{x} = 1$
   (v) $\lim_{x \to 0}\frac{a^x - 1}{x} = \ln a$, $a > 0$, $a \ne 1$
   (vi) $\lim_{x \to 0}(1 + x)^{\frac{1}{x}} = e$
   (vii) $\lim_{x \to a}(1 + f(x))^{g(x)} = e^{\lim_{x \to a} f(x) \cdot g(x)}$
   If $f(x) \to 0$ and $g(x) \to \infty$ as $x \to a$
3. **De' L'Hospital rule :**
   If $\lim_{x \to a}\frac{f(x)}{g(x)}$ is in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ form then L' Hospital rule is applicable. We differentiate numerator and denominator separately to evaluate the limit.

Continuity

(a) Let $f(x)$ be a real valued function and $a \in R$, then function $f(x)$ is said to be continuous at $x = a$ if
   (i) $f(a)$ is defined
   (ii) $\lim_{x \to a} f(x)$ exists

   (iii) $\lim_{x \to a} f(x) = f(a)$
   Note :
   (i) Also, we can say that, a function $f(x)$ is said to be continuous if $\lim_{x \to a} f(x) = f(a)$.
   (ii) If any one of the above three conditions is not satisfied then $f(x)$ is discontinuous at $x = a$.
   (iii) Geometrically, if we draw the graph of a function without raising pen or pencil, then we say that it is a continuous function.
(b) Continuity on an interval
   (i) Open interval : A function $f(x)$ is said to be continuous on (a, b) if it is continuous at each point of (a, b).
   (ii) Closed interval : A function $f(x)$ is said to be continuous on $[a, b]$ if
   (a) It is continuous on (a, b)
   (b) $\lim_{x \to a^+} f(x) = f(a)$
   (c) $\lim_{x \to b^-} f(x) = f(b)$
4. **Continuity of composite function :** If $f(x)$ is continuous at $x = c$ and $g(x)$ is continuous at $x = f(c)$, then $g(f(x))$ is continuous at $x = c$.
   Let $f(x) = x\sin x$ and $g(x) = |x|$ are continuous at $x = 0$
   Thus the function $g(f(x)) = |x\sin x|$ will also be continuous at $x = 0$
5. **Intermediate value theorem**
   Let $f(x)$ be a continuous real valued function and $a, b \in R$.
   (i) If $f(a)$ and $f(b)$ are of opposite signs, then there exists at least one solution of the equation $f(x) = 0$ in (a, b)
   (ii) If k is any real number between $f(a)$ and $f(b)$ then the exists at least one solution of the equation $f(x) = k$ in (a, b)
6. **Differentiability**
   Let $f(x)$ be a real valued function and $a \in R$. A function $f(x)$ is said to be differentiable at $x = a$ if $f'(a)$ exists
   i.e. $\lim_{h \to 0}\frac{f(a+h) - f(a)}{h} = \lim_{h \to 0}\frac{f(a-h) - f(a)}{-h}$
   i.e. L.H.D. = R.H.D.
   Thus, $f'(a) = \lim_{h \to 0}\frac{f(a+h) - f(a)}{h}$
7. **Differentiability on an interval :** Let $f(x)$ be a real valued function and $a, b \in R$
   (i) Open interval : A function $f(x)$ is said to differentiable on (a, b) if it is differentiable to each point of (a, b).
   (ii) Closed interval : A function $f(x)$ is said to be a differentiable on $[a, b]$ if

   (a) It is differentiable on (a, b)
   (b) $\lim_{h \to 0}\frac{f(a+h) - f(a)}{h}$ is finite
   $\lim_{h \to 0}\frac{f(b-h) - f(b)}{-h}$ is finite
   (c) $\lim_{x \to a}\frac{f(x) - f(a)}{x - a} = f'(b)$

   Note :
   1. Geometrically, a function $f(x)$ is said to be differentiable at $x = a$ iff there exists a unique tangent of finite slope at $x = a$
   2. A function $f(x)$ is not differentiable at $x = a$ iff there is a corner point at $x = a$.

Chap-14 - Applications of Derivatives

Formulae Chart

1. The equation of the tangent to a curve at (x_0, y_0)
   $y - y_0 = m(x - x_0)$
   The equation of the normal to a curve at (x_0, y_0)
   $y - y_0 = -\frac{1}{m}(x - x_0)$
   where $m = \left.\frac{dy}{dx}\right|_{(x_0, y_0)}$
2. The angle of intersection of two curves
   $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$
   where m_1, m_2 are the slope of the tangents to the curves at the point of intersection.
3. Increasing and decreasing functions :
   ❖ If $f(x_2) > f(x_1)$, $\forall x_2 > x_1$, $x_1, x_2 \in D_f$ then $f(x)$ is strictly increasing.
   ❖ If $f'(x) > 0$, $\forall x \in D_f$, then $f(x)$ is strictly increasing function.
   ❖ If $f(x_2) \ge f(x_1)$, $\forall x_2 > x_1$, $x_1, x_2 \in D_f$, then $f(x)$ is increasing function.
   ❖ If $f'(x) \ge 0$, $\forall x \in D_f$, then $f(x)$ is increasing function.
   ❖ If $f(x_2) < f(x_1)$, $\forall x_2 > x_1$, $x_1, x_2 \in D_f$, then $f(x)$ is strictly decreasing function.
   ❖ If $f'(x) < 0$, $\forall x \in D_f$, then $f(x)$ is strictly decreasing function.
   ❖ If $f(x_2) \le f(x_1)$, $\forall x_2 > x_1$, $x_1, x_2 \in D_f$, then $f(x)$ is decreasing function.
   ❖ If $f'(x) \le 0$, $\forall x \in D_f$, then $f(x)$ is decreasing function.
4. **Maxima and Minima**
   ❖ If $f(x_0 + h) > f(x_0)$ and $f(x_0 - h) > f(x_0)$, then $f(x)$ has a relative minima at $x = x_0$.
   ❖ If $f(x_0 + h) < f(x_0)$ and $f(x_0 - h) < f(x_0)$, then $f(x)$ has a relative maxima at $x = x_0$.
   ❖ If $f(x)$ is differentiable function, then for maxima or minima $f'(x) = 0 \Rightarrow x = x_1, x_2, \ldots, x_n$.
   ❖ **First derivative test :** If f' changes its sign from positive to negative about a critical point then it has point of local maxima.
   ❖ If f' changes sign from negative to positive then f has a point of minima.
   ❖ If there is no sign change about the critical point then f has neither maxima nor minima at that point.
   ❖ n^{th} derivative test : Solve $f'(x) = 0 \Rightarrow x = x_1, x_2, \ldots, x_n, \ldots$
   ❖ If $f''(x_0) > 0$, then $f(x)$ has relative minima at $x = x_0$
   ❖ If $f''(x_0) < 0$, then $f(x)$ has relative maxima at $x = x_0$.
   ❖ If $f''(x_0) = 0$, find $f'''(x)$.

   ❖ If $f'''(x_0) \ne 0 \Rightarrow$ neither maxima nor minima it is called point of inflexion.
   ❖ If $f'''(x_0) = 0$, find $f^{iv}(x)$.
   Repeat the process till we get $f^{(n)}(x_0) \ne 0$.
   If n is odd $\Rightarrow$ neither maxima nor minima
   If n is even $\Rightarrow$ $f^{(n)}(x_0) > 0 \Rightarrow$ minima at $x = x_0$
   $f^{(n)}(x_0) < 0 \Rightarrow$ maxima at $x = x_0$
5. **Rolle's theorem :**
   Let f be a real valued function which is
   (i) Continuous on bounded and closed interval $[a, b]$
   (ii) Differentiable at every point x in the open interval (a, b)
   (iii) $f(a) = f(b)$
   then there exists at least one point c in (a, b)
   such that $f'(c) = 0$
6. **Lagrange's Mean Value Theorem :** Let f be a real valued function which is
   (i) Continuous on $[a, b]$
   (ii) Differentiable in (a, b)
   Then there exists at least one value of x, say c, such that
   $f'(c) = \frac{f(b) - f(a)}{b - a}$, $c \in (a, b)$

Chap-15 - Indefinite Integrals

Formulae Chart

1. **Integration by substitution :**
   Basic rule of substitution :
   $\int f(x)dx = \int f[\phi(z)] \cdot \phi'(z)dz$
   Substitute $x = \phi(z)$
2. **Integration by parts :**
   If u and v be two functions of x then $\int uv\,dx = u\int v\,dx - \int\left(\frac{du}{dx}\int v\,dx\right)dx$ is known as formula for integration by parts.
   Note : In finding integrals by this method proper choice of u and v is essential. In general the function as u is taken which comes first in the word ILATE.
   where, I - *Inverse circulated function*
   L - *Logarithmic function*
   A - *Algebraic function*
   T - *Trigonometric function*
   E - *Exponential function*
   Note :
   * *If both the functions are trigonometric, take that function as v whose integral is simpler.*
   ** *If both the functions are algebraic take that function as 'u' whose d.c. is simpler.*
3. **Integration of rational function :**
   If integrand is of the form $\frac{P(x)}{Q(x)}$, where $P(x)$ and $Q(x)$ are polynomials of x.
   Case I. If $\frac{P(x)}{Q(x)}$ is proper rational function (i.e. highest power of x in $Q(x) > P(x)$)
   Integration by partial fraction is done.
   Case II. If $\frac{P(x)}{Q(x)}$ is improper rational function (i.e. highest power of x in $(Q(x) < P(x))$
   Then it is made proper rational function by simple division and then partial fraction is used
   i.e. $\frac{P(x)}{Q(x)} = \frac{f(x) \cdot Q(x) + h(x)}{Q(x)} = f(x) + \frac{h(x)}{Q(x)}$
   $f(x)$ is integrated by general rule and $\frac{h(x)}{Q(x)}$ is integrated by partial fraction.
4. **Integration of certain irrational expressions :**
   (i) If the integrand is a rational function of fractional powers of an independent variable x i.e.

   the function $R\left(x, x^{p_1/q_1}, \ldots, x^{p_k/q_k}\right)$. Substitute $x = t^m$, m is L.C.M. of $q_1, q_2, \ldots, q_k$.
   (ii) If the integrand is a rational function of x and fractional powers of a linear fractional function of the form $\frac{ax+b}{cx+d}$ i.e. $R\left(x, x^{p_1/q_1}, \ldots, \left(\frac{ax+b}{cx+d}\right)^{p_1/q_1}, \left(\frac{ax+b}{cx+d}\right)^{p_2/q_2}, \ldots, \left(\frac{ax+b}{cx+d}\right)^{p_k/q_k}\right)$,
   substitute $\frac{ax+b}{cx+d} = t^m$ where m is L.C.M. of $(\beta_1, \beta_2, \ldots, \beta_k)$.
5. **Integration of the form** $\int\sqrt{ax^2 + bx + c}$
   (i) Integrals of the form $I = \int\frac{Mx + N}{\sqrt{ax^2 + bx + c}}dx$
   Substitute $x + \frac{b}{2a} = t$ and reduce it to the form $I = M_1\int\frac{tdt}{\sqrt{at^2 + k}} + N_1\int\frac{dt}{\sqrt{at^2 + k}}$, M_1, N_1, k are new constants.
   (ii) Integrals of the form $I = \int\frac{P_m(x)}{\sqrt{ax^2 + bx + c}}dx$
   $P_m(x)$ is polynomial of degree m
   $I = P_{m-1}(x)\sqrt{ax^2 + bx + c} + k\int\frac{dx}{\sqrt{ax^2 + bx + c}}$
   where $P_{m-1}(x)$ is polynomial of degree $(m-1)$ and k is constant.
   Coefficient of $P_{m-1}(x)$ is determined by the method of undetermined coefficients.
   (iii) Integrals of the form $\int\frac{dx}{(x - \alpha_1)^m\sqrt{ax^2 + bx + c}}$
   Substitute $(x - \alpha_1) = \frac{1}{t}$
   Note : *For the integration of the form* $\int\sqrt{ax^2 + bx + c}\,dx$ or $\int\frac{dx}{\sqrt{ax^2 + bx + c}}$ or $\int\frac{dx}{ax^2 + bx + c}$
   1st convert $ax^2 + bx + c$ in the form $A^2 + x^2$, $x^2 - A^2$ or $A^2 - x^2$, where A is constant and x is $px + q$.
   Following results is used :
   (i) $\int\frac{dx}{\sqrt{x^2 \pm a^2}} = \log\left|x + \sqrt{x^2 \pm a^2}\right| + C$
   (ii) $\int\sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$
   (iii) $\int\sqrt{x^2 \pm a^2}dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\log\left(x + \sqrt{x^2 \pm a^2}\right) + C$
   (iv) $\int\frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + C$

6. **Integration of the form of** $\int x^m(a + bx^n)^p\,dx$**, m, n, p are rational numbers.**
   Case 1. If P is a positive integer then integrand is expanded by binomial expansion.
   Case 2. If P is negative integer substitute $x = t^k$ where k is L.C.M. of denominator of m and n.
   Case 3. $\frac{m+1}{n}$ is an integer. Put $a + bx^n = t^\alpha$ where α is denominator of the fraction P.
   Case 4. $\frac{m+1}{n} + P$ is an integer, put $a + bx^n = t^\alpha x^n$ where α is denominator of fraction P.
7. **Integration of trigonometric function.**
   (i) Integration of the form $\int\sin^m x \cdot \cos^n x\,dx$
   (a) m = odd positive integer, substitute $z = \cos x$.
   (b) n = odd positive integer, substitute $z = \sin x$.
   (c) If both m and n are odd positive integer put $z = \sin x$ or $\cos x$.
   (d) If m and n both are not odd positive integer
   (1) If $m + n$ is even negative integer put $z = \tan x$.
   (2) If $(m + n)$ is even positive integer, express the integrands as the algebraic sum of sines and cosines of multiple angles using formula
   $\sin^2 x = \frac{1 - \cos 2x}{2}$, $\cos^2 x = \frac{1 + \cos 2x}{2}$, $\sin^3 x = \frac{3\sin x - \sin 3x}{4}$, $\cos^3 x = \frac{3\cos x + \cos 3x}{4}$
   (ii) Integral of the form $\int\tan^m x \cdot \sec^n x\,dx$ or $\int\cot^m x \cdot \operatorname{cosec}^n x\,dx$
   (a) If m is even positive integer, put $z = \tan x$ in I_1, $z = \cot x$ in I_2
   (b) If m is even negative integer then put $\sec^2 x - 1$ in place of $\tan^2 x$ in I_1 and $\operatorname{cosec}^2 x - 1$ in place of $\cot^2 x$ in I_2 and substitute $z = \tan x$ (in I_1)
   $= \cot x$ (in I_2)
   (c) $m = 0$, n is odd positive integer greater than 1 then method of integration by parts is used
   Ex. $\int\operatorname{cosec}^3 x\,dx$, $\int\sec^3 x\,dx$
   (iii) If integrand contains any expression of the form $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, $\operatorname{cosec}\theta$, where θ is function of x. [Ex. $\int\sin\sqrt{x}\,dx$], put $\theta = z$.
   (iv) Integration of the form $\int\frac{a\sin x}{a\sin x + b\cos x}dx$, $\int\frac{\cos x}{a\sin x + b\cos x}dx$ or $\int\frac{p\sin x + q\cos x}{a\sin x + b\cos x}dx$
   where $a \ne 0$, $b \ne 0$. Put numerator = A (Denominator) + B(D.C. of denominator)
   (v) Integration of the form $\int\frac{dx}{a\cos^2 x + 2b\sin x \cdot \cos x + c\sin^2 x}$, $\int\frac{dx}{a + b\cos^2 x}$ or $\int\frac{dx}{a + b\sin^2 x}$. Divide numerator and denominator by $\cos^2 x$ and put $\tan x = z$.

   (vi) Integration of the form
   (i) $\int\frac{dx}{a\cos x + b\sin x + c}$ (ii) $\int\frac{dx}{a + b\cos x}$ (iii) $\int\frac{dx}{a + b\sin x}$
   Write $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$, $\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$ and substitute $z = \tan\frac{x}{2}$.
   (vii) Trigonometric substitution, when integrand involves expression of the form
   (a) $\sqrt{a^2 - x^2}$, substitute $x = a\cos\theta$ or $a\sin\theta$.
   (b) $a^2 + x^2$ or $\sqrt{a^2 + x^2}$, substitute $x = a\tan\theta$ or $a\cot\theta$.
   (c) $\sqrt{x^2 - a^2}$, $x = a\sec\theta$ or $a\operatorname{cosec}\theta$
   (d) $\sqrt{\frac{a - x}{a + x}}$ or $\sqrt{\frac{a + x}{a - x}}$, $x = a\cos 2\theta$.
   (e) $\sqrt{\frac{x}{a - x}}$ or $\sqrt{\frac{a - x}{x}}$, $x = a\sin^2\theta$ or $a\cos^2\theta$
   (f) $\sqrt{\frac{x}{a + x}}$ or $\sqrt{\frac{a + x}{x}}$, $x = a\tan^2\theta$ or $a\cot^2\theta$
   (g) $\sqrt{\frac{a - x}{x - b}}$ or $\sqrt{\frac{x - b}{a - x}}$ or $\sqrt{(a - x)(x - b)}$, substitute $x = a\cos^2\theta + b\sin^2\theta$
8. **Integration of the form** $\int e^x[f(x) + f'(x)]\,dx$
   $\int e^x[f(x) + f'(x)]\,dx = e^x f(x) + C$
9. **Integrals of the form**
   (a) $\int\frac{px + q}{ax^2 + bx + c}dx$ (b) $\int\frac{px + q}{\sqrt{ax^2 + bx + c}}dx$
   (c) $\int(px + q)\sqrt{ax^2 + bx + c}\,dx$ (d) $\int\frac{px^2 + qx + r}{ax^2 + bx + c}dx$
   for (a), (b), (c), write $px + q$ = A (d.c. of $(ax^2 + bx + c)$) + B
   By equating similar power of x, find A and B.
   For (d) $px^2 + qx + r = A(ax^2 + bx + c) + B(2ax + b) + c$ find A, B, C, and solve.
10. **Integral of the form** $\int\frac{dx}{X\sqrt{Y}}$
    Use substitution as given in table

X	Y	Substitution
Linear	Linear	$Z^2 = Y$
Quadratic	Linear	$Z^2 = Y$
Linear	Quadratic	$Z = \frac{1}{X}$
Quadratic Quadratic	$Z^2 = \frac{1}{Y}$	or $Z = \frac{1}{X}$

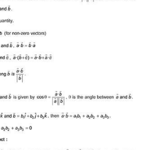
Chap-16 - Definite Integrals and their Applications

Formulae Chart

1. If $F(x)$ is the antiderivative of $f(x)$ then
   $\int_a^b f(x)dx = [F(x) + c]_a^b$
   $= F(b) - F(a)$
   this is called fundamental theorem of integral calculus.
2. Properties of definite integral :
   (i) $\int_a^b f(x)\,dt = \int_a^b f(t)\,dt$
   (ii) $\int_a^b f(x)dx = -\int_b^a f(x)dx$
   (iii) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, $a < c < b$ (this property is also true when c doesn't lie between a & b)
   (iv) $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$
   (v) $\int_{-a}^{a} f(x)dx = \begin{cases} 0, & \text{if } f(x) \text{ is an odd function} \\ 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is an even function} \end{cases}$
   (vi) $\int_0^{2a} f(x)dx = \int_0^a [f(x) + f(2a - x)]dx$
   $\Rightarrow \int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(2a - x) = f(x) \\ 0, & \text{if } f(2a - x) = -f(x) \end{cases}$
   (vii) $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)dt = f[\psi'(x)] \cdot \psi'(x) - f[\phi(x)] \cdot \phi'(x)$ this is called Newton - Leibnitz rule.
   (viii) $\int_0^a f(x)dx = \int_0^a f(x - c)dx$
   (ix) $\int_0^a f(x)dx = \frac{1}{c}\int_0^{ac} f\left(\frac{x}{c}\right)dx$

   (x) If f is periodic function in properties x to xiii with period T, then
   $\int_0^{nT} f(x)dx = n\int_0^T f(x)dx$, $n \in N$
   (xi) $\int_a^{a+T} f(x)dx$, $(a > 0)$ is independent of 'a'
   $\therefore$ we can write
   $\int_a^{a+T} f(x)dx = \int_0^T f(x)dx$
   (xii) $\int_a^{a+nT} f(x)dx = n\int_0^T f(x)dx$, $n \in N$
   (xiii) $\int_{mT}^{nT} f(x)dx = (n - m)\int_0^T f(x)dx$, $m, n \in N$
   (a) $\int_{a+mT}^{a+nT} f(x)dx = (n - m)\int_0^T f(x)dx$, $m, n \in N$
   (b) $\int_{a+nT}^{b+nT} f(x)dx = \int_a^b f(x + nT)dx = \int_a^b f(x)dx$
   (xiv) $\int_a^b f(x)dx = (b - a)\int_0^1 f((b - a)x + a)dx$
   (xv) $\frac{d}{dx}\int_a^b f(x, t)dt = \int_a^b\frac{\partial}{\partial x}[f(x, t)]dt$ where $\frac{\partial}{\partial x}[f(x, t)]$ is derivative of f w.r.t. 'x' keeping 't' as constant.
   (xvi) $\frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(x, t)dt = f[x, \psi(x)] \cdot \psi'(x) - f[x, \phi(x)] \cdot \phi'(x) + \int_{\phi(x)}^{\psi(x)}\frac{\partial}{\partial x}f(x, t)dt$
3. Estimation of integral :
   (i) $\left|\int_a^b f(x)dx\right| \le \int_a^b |f(x)|dx$
   (ii) If $g(x) < f(x) < h(x)$ is $[a, b]$, then $\int_a^b g(x)dx < \int_a^b f(x)dx < \int_a^b h(x)dx$
   (iii) (a) If f is an increasing function and has concave up graph in $[a, b]$ then
   $(b - a)f(a) < \int_a^b f(x)dx < b - a\frac{f(b) + f(a)}{2}$
   (b) If f is an increasing function and has concave down (or convex up) graph is $[a, b]$, then

   (iv) If M and m be the greatest and least values of function f on $[a, b]$, then,
   $m(b - a) \le \int_a^b f(x)dx \le M(b - a)$
   (v) If 'f' is a continuous function is $[a, b]$, then it assumes its mean value is $[a, b]$, that is
   $\frac{1}{b - a}\int_a^b f(x)dx = f(c)$, for same c, such that $a \le c \le b$
   This is known as mean value theorem of integral calculus.
   (vi) $\int_a^b f(x)g(x)dx \le \sqrt{\left[\int_a^b f^2(x)dx\right]\left[\int_a^b g^2(x)dx\right]}$
   This is known as Schwarz - Bunyakovsky inequality.
4. Definite integral as the limit of a sum :
   If $f(x)$ is continuous function on $[a, b]$, then
   $\int_a^b f(x)dx = \lim_{h \to 0} h\sum_{r=0}^{n-1} f(a + rh)$ where $nh = b - a$, $n \to \infty$ as $h \to 0$
   $\int_0^1 f(x)dx = \lim_{n \to \infty}\frac{1}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)$
5. Area under the curve or curves :
   (i) $A = \int_a^b f(x)dx$



   (ii) $A = \int_a^b [f(x) - g(x)]dx$

Chap-17_Differential Equation

Formulae Chart

1. A differential equation of type $\frac{dy}{dx} = F(x, y)$ is homogeneous differential equation if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$.
2. To solve homogeneous differential equation we put
   $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$
   or $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$
3. $\frac{dy}{dx} + Py = Q$ where P, Q are functions of x alone or constants is linear differential equation.
   I.F. $= e^{\int P dx}$
   And the solution is given by
   $y \cdot (\text{I.F.}) = \int Q(\text{I.F.})dx + c$
4. $\frac{dx}{dy} + Px = Q$ where P, Q are the functions of y only or constants
   I.F. $= e^{\int P dy}$
   And solution is $x(\text{I.F.}) = \int Q(\text{I.F.})dy + c$.

Chap-18_Vector Algebra

Formulae Chart

1. **Vectors when co-ordinates of end points are given :** If $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ be the end-points of line segment AB, then $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$.
   $(x_2 - x_1)$, $(y_2 - y_1)$ and $(z_2 - z_1)$ are components of $\overrightarrow{AB}$ along x, y and z-axis respectively. These are also direction ratios along x, y and z-axis respectively.
2. **Section formula :** Position vector $\vec{r}$ of a point which divides the join of two given points with position vector $\vec{a}$ and $\vec{b}$ in the ratios $l : m$ internally is $\vec{r} = \frac{l\vec{b} + m\vec{a}}{l + m}$.
   Externally, $\vec{r} = \frac{l\vec{b} - m\vec{a}}{l - m}$
3. **Scalar product of two vectors :**
   (i) The scalar product of two vector $\vec{a}$ and $\vec{b}$ defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.
   (ii) $\vec{a} \cdot \vec{b}$ is a scalar quantity.
   (iii) $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$ (for non-zero vectors)
   (iv) For two vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
   (v) For vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
   (vi) Projection of $\vec{a}$ along $\vec{b}$ is $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
   (vii) Angle between $\vec{a}$ and $\vec{b}$ is given by $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$, θ is the angle between $\vec{a}$ and $\vec{b}$.
   (viii) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, then $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.
   If $\vec{a} \perp \vec{b} \Rightarrow a_1b_1 + a_2b_2 + a_3b_3 = 0$
4. **Vector or cross product :**
   (i) $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\hat{n}$, θ is the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is a unit vector perpendicular to $\vec{a}$ and $\vec{b}$ and direction is such that $\vec{a}$, $\vec{b}$ and $\hat{n}$ form a right hand system.

   (ii) $\vec{a} \times \vec{b}$ is a vector quantity whose magnitude is $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$.
   (iii) If θ is angle between $\vec{a}$ and $\vec{b}$, then $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
   (iv) If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$.
   Then $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
   (v) Unit vector $\hat{n}$ perpendicular to $\vec{a}$ and $\vec{b}$ is $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.
5. **Scaler Triple product :**
   $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$
   $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$
   $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$
   $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}] = [\vec{b}\ \vec{c}\ \vec{a}] = [\vec{c}\ \vec{a}\ \vec{b}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
   (i) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar then $[\vec{a}\ \vec{b}\ \vec{c}] = 0$
   (ii) $\vec{a}$, $\vec{b}$, $\vec{c}$ are coterminous edges of parallelepiped then volume of parallelepiped $= [\vec{a}\ \vec{b}\ \vec{c}]$.
   (iii) $\vec{a}$, $\vec{b}$, $\vec{c}$ are coterminous edges of tetrahedrons than volume of tetrahedrons $= \frac{1}{6}[\vec{a}\ \vec{b}\ \vec{c}]$.
6. **Vector Triple product :**
   $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$
   If gives a vector orthogonal to $\vec{a}$ and coplanar to $\vec{b}$ and $\vec{c}$.

Chap-19 3-D Geometry

Formulae Chart

1. **Shortest distance between for parallel lines**
   $\vec{r} = \vec{a} + \lambda\vec{b}$
   $\vec{r} = \vec{c} + \mu\vec{b}$
   is $\left|\frac{(\vec{a} - \vec{c}) \times \vec{b}}{|\vec{b}|}\right|$
2. **Shortest distance between two skew lines**
   $\vec{r} = \vec{a} + \lambda\vec{b}$
   $\vec{r} = \vec{c} + \mu\vec{d}$
   is $\left|\frac{(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})}{|\vec{b} \times \vec{d}|}\right|$
3. **Condition for two lines to be coplanar or intersecting**
   $(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{a}) = 0 \Rightarrow [\vec{c} - \vec{a}\ \vec{b}\ \vec{d}] = 0$
4. **Angle between line and plane**
   $L : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$
   $P : a_2x + b_2y + c_2z + d_2 = 0$
   $\sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$
   Line and plane are parallel if $a_1a_2 + b_1b_2 + c_1c_2 = 0$
   Line and planes are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
5. **Foot of perpendicular of point (α, β, γ) with respect to $ax + by + cz + d = 0$**
   $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} = \frac{a\alpha + b\beta + c\gamma + d}{a^2 + b^2 + c^2}$, (x_1, y_1, z_1) is foot of perpendicular.
   Image of point (α, β, γ) with respect to $ax + by + cz + d = 0$
   $\frac{\alpha - x_2}{a} = \frac{\beta - y_2}{b} = \frac{\gamma - z_2}{c} = \frac{2(a\alpha + b\beta + c\gamma + d)}{a^2 + b^2 + c^2}$, (x_2, y_2, z_2) is image.

Chap-20 Probability

Formulae Chart

(i) Let S be a sample space and E be an event, then the probability of E, denoted by P(E) is defined as $P(E) = \frac{n(E)}{n(S)}$.

(ii) In case of some finite geometrical measurement such as length, area or volume, the probability of an event is defined as $P(E) = \frac{\text{Length of E}}{\text{Length of S}}$ or $\frac{\text{Area of E}}{\text{Area of S}}$ or $\frac{\text{Volume of E}}{\text{Volume of S}}$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
If A, B and C are mutually exclusive events, then
$P(A \cup B) = P(A) + P(B)$ and $P(A \cap B) = 0$
$P(A \cup B \cup C) = P(A) + P(B) + P(C)$, $P(A \cap B \cap C) = 0$

(iv) **Conditional Probability**
If A and B are dependent events, then the probability of the event B when the event A has already occured is called conditional probability denoted by $P\left(\frac{B}{A}\right)$ is defined as
$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$, $P(A) \ne 0$.
Similarly $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$, $P(B) \ne 0$.

(v) **Total Probability**
Let $E_1, E_2, E_3, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment, then the probability of the occurence of any event E, which occurs with E_1 or E_2 or E_3 or …… or E_n is given by $P(E) = P(E_1)P\left(\frac{E}{E_1}\right) + P(E_2)P\left(\frac{E}{E_2}\right) + \ldots + P(E_n)P\left(\frac{E}{E_n}\right)$

(vi) **Baye's Theorem**
Let $E_1, E_2, \ldots, E_n$ be n mutually exclusive and exhaustive events associated with a random experiment and E is any other event which occurs with E_1 or E_2 or E_3 …… or E_n, then probability of the occurence of the event E_1 or E_2 or …… or E_n when E has already occured is given by

$$P\left(\frac{E_k}{A}\right) = \frac{P(E_k)P\left(\frac{A}{E_k}\right)}{\sum_{k=1}^{n} P(E_k)P\left(\frac{A}{E_k}\right)}, k = 1, 2, 3, \ldots, n$$

(vii) **Probabilities using Binomial Theorem**
   (a) Probability of k success in n trials
   $P(k) = {}^nC_k\, p^kq^{n-k}$, $k \le n$
   (b) Probability of at least k success in n trials
   $P(\ge k) = \sum_{r=k}^{n} {}^nC_r\, p^rq^{n-r}$
   (c) Probability of atmost k success in n trials
   $P(\le k) = \sum_{r=0}^{k} {}^nC_r\, p^rq^{n-r} = {}^nC_0\, p^0q^n + {}^nC_1\, pq^{n-1} + \ldots + {}^nC_k\, p^kq^{n-k}$
   (d) Probability of atleast one success
   $= 1 -$ Probability of number success $= 1 - {}^nC_0\, q^n$.

(viii) The mean, variance and standard deviation of binomial distribution are np, npq and $\sqrt{npq}$ respectively where
   n = Number of trials
   p = Probability of success (Occurence)
   q = Probability of failure (non-occurence)

(ix) The probability of selecting exactly one pair of shoes when $2r$ shoes are to be selected from n pairs of shoes is $\frac{n \cdot {}^{n-1}C_{2r-2} \cdot 2^{2r-2}}{{}^{2n}C_{2r}}$.

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